Influence of heterogeneity on the interpretation of pumping test data in leaky aquifers

Nadim K. Copty,[1] Paolo Trinchero,[2] Xavier Sanchez-Vila,[2] Murat Savas Sarioglu,[1] and Angelos N. Findikakis[3]

Received 28 April 2008; revised 9 September 2008; accepted 17 September 2008; published 15 November 2008.

[1] Pumping tests are routinely interpreted from the analysis of drawdown data and their derivatives. These interpretations result in a small number of apparent parameter values which lump the underlying heterogeneous structure of the aquifer. Key questions in such interpretations are (1) what is the physical meaning of those lumped parameters and (2) whether it is possible to infer some information about the spatial variability of the hydraulic parameters. The system analyzed in this paper consists of an aquifer separated from a second recharging aquifer by means of an aquitard. The natural log transforms of the transmissivity, ln *T*, and the vertical conductance of the aquitard, ln *C*, are modeled as two independent second-order stationary spatial random functions (SRFs). The Monte Carlo approach is used to simulate the time-dependent drawdown at a suite of observation points for different values of the statistical parameters defining the SRFs. Drawdown data at each observation point are independently used to estimate hydraulic parameters using three existing methods: (1) the inflection-point method, (2) curve-fitting, and (3) the double inflection-point method. The resulting estimated parameters are shown to be space dependent and vary with the interpretation method since each method gives different emphasis to different parts of the time-drawdown data. Moreover, the heterogeneity in the pumped aquifer or the aquitard influences the estimates in distinct manners. Finally, we show that, by combining the parameter estimates obtained from the different analysis procedures, information about the heterogeneity of the leaky aquifer system may be inferred.

Citation: Copty, N. K., P. Trinchero, X. Sanchez-Vila, M. S. Sarioglu, and A. N. Findikakis (2008), Influence of heterogeneity on the interpretation of pumping test data in leaky aquifers, *Water Resour. Res.*, *44*, W11419, doi:10.1029/2008WR007120.

1. Introduction

[2] In the analysis of pumping tests, drawdown data as function of space and/or time are normally used to infer representative hydraulic parameters of the perturbed aquifer volume. Most existing pumping test analysis procedures assume that the aquifer system is homogeneous, or at most consists of a few homogeneous deterministically located units. Under such assumptions of homogeneity, it is possible to devise graphical or analytical methods that can provide the exact parameters of the system if noise-free drawdown data are available (for example, *Theis* [1935] for confined aquifers, *Hantush and Jacob* [1955] for leaky aquifers).

[3] A salient characteristic of natural geologic formations is that they are heterogeneous with complex patterns of spatial variability. As a result, significant effort has been devoted over the past four decades to address the problem of radially convergent flow toward a pumping well in heterogeneous aquifers. A number of review papers have been published on this topic [e.g., *Renard and de Marsily*, 1997; *Rubin*, 2003; *Raghavan*, 2004; *Sanchez-Vila et al.*, 2006]. Some of the more relevant studies are summarized next.

[4] A number of published papers focused on the potential existence and determination of an effective hydraulic conductivity value, defined as the negative ratio between the expected values of the flow and the hydraulic gradient at any given point [e.g., *Dagan*, 1982, 1989; *Gomez-Hernandez and Gorelick*, 1989; *Naff*, 1991; *Neuman and Orr*, 1993; *Indelman and Abramovich*, 1994; *Indelman et al.*, 1996; *Sanchez-Vila*, 1997; *Riva et al.*, 2001; *Dagan*, 2001; *Guadagnini et al.*, 2003; *Indelman*, 2003a, 2003b]. When the effective hydraulic conductivity is constant throughout the domain, it can be considered an intrinsic property of the medium. However, for radially convergent flow, it was shown as early as the study of *Shvidler* [1962] that such a constant effective value does not exist. Rather, effective hydraulic conductivity increases from the harmonic mean near the pumping well to the geometric mean at some distance from the well, and this distance depends on the correlation length of the hydraulic conductivity field [*Indelman and Abramovich*, 1994; *Sanchez-Vila*, 1997]. The effective hydraulic parameters are functions also of the flow dimensionality and time [*Dagan*, 1982].

[1]Institute of Environmental Sciences, Bogazici University, Bebek, Istanbul, Turkey.
[2]Department of Geotechnical Engineering and Geosciences, Technical University of Catalonia, Barcelona, Spain.
[3]Bechtel National Inc., San Francisco, California, USA.

0043-1397/08/2008WR007120

[5] Another group of papers focus on the estimation of an upscaled hydraulic conductivity that is averaged over a finite volume for an individual realization [e.g., *Gomez-Hernandez and Gorelick*, 1989; *Oliver*, 1990; *Desbarats*, 1992; *Durlofsky*, 2000; *Rubin*, 2003; *Copty et al.*, 2006]. The upscaled hydraulic conductivity is a local, non-unique estimate, dependent on the flow conditions. In confined aquifers the upscaled conductivity can be written as a weighted-average of the point values, with the weights decreasing with increasing distance from the pumping well [*Sanchez-Vila et al.*, 1999a].

[6] More recently, some researchers examined the problem of defining the statistical spatial structure from the drawdown data [*Copty and Findikakis*, 2004a; *Neuman et al.*, 2004, 2007; *Firmani et al.*, 2006]. These studies show that the variance and integral scale of the hydraulic conductivity or transmissivity may be inferred when a large number of pumping tests and observation points are available.

[7] While all the papers cited so far provide a general framework for the analysis of radially convergent flow in heterogeneous media, in practice most pumping tests are routinely interpreted by simple methods based on a homogenized conception of the system. The interpreted parameters (transmissivity and storativity in confined aquifers, plus the leakage factor in leaky aquifers) lump all the underlying heterogeneity, providing some averaged value of the spatially dependent parameters. Although this is a commonly encountered problem, relatively few papers can be found that assess the impact of the heterogeneity of the hydraulic parameters on the interpretation of pumping tests. Earlier studies such as those of *Barker and Herbert* [1982], *Butler* [1988], and *Butler and Liu* [1993] examined this problem for systems with deterministically defined high or low permeable inclusions embedded in a homogeneous background *T* field. *Serrano* [1997] developed an analytic solution that relates the mean and variance of the transmissivity of confined aquifers to the mean and variance of the drawdown. *Meier et al.* [1998] and *Sanchez-Vila et al.* [1999b] evaluated the validity of the Cooper-Jacob method [*Cooper and Jacob*, 1946] for the interpretation of time-drawdown data in aquifers with heterogeneous transmissivity and uniform storativity. They showed that the transmissivity estimated using the Cooper-Jacob method does not depend on the observation point location, with a value close to the geometric mean of the transmissivity distribution. However, the estimated storativity could significantly vary with the location of the observation point, which is indicative of the flow point-to-point connectivity between the pumping well and the observation point [see also *Trinchero et al.*, 2008b]. *Copty and Findikakis* [2004a, 2004b] used a Monte Carlo approach to study the effect of spatial heterogeneity on the transient drawdown due to pumping in a confined aquifer, and on the conditions where the Cooper-Jacob assumptions would be valid. *Wu et al.* [2005] used the Thies method to estimate flow parameters from time-drawdown data at a single observation well in a highly heterogeneous aquifer with log-Gaussian transmissivity and storativity fields. There results show that the estimated transmissivity and storativity values estimated from the early drawdown data change with time. The storativity estimate stabilize rather quickly to a value that is function of the storativity of the aquifer volume between the pumping and the observation wells. In contrast the estimated transmissivity at late times is function of the entire flow domain, and approach a value that is close but not equal the geometric mean. *Knight and Kluitenberg* [2005] derived explicit analytic expressions for the sensitivity of pumping tests and slug tests to variations in the transmissivity and storativity, for the case when the pumping well and observation well are collocated and when the two wells are at different locations. *Leven and Dietrich* [2006] used sensitivity coefficients to study the effect of spatial variability of the transmissivity and storativity of confined aquifer on the interpretation of single-well and two-well pumping tests.

[8] Almost all the studies cited above have focused on the problem of radially convergent flow in heterogeneous confined aquifers. However, in natural geologic systems, confining layers overlying and/or underlying an aquifer often leak. Aquifer-aquitard systems are found worldwide, particularly in multi-layer and more complex geologic formations such as alluvial aquifers. In such systems, drawdown values depend on the hydraulic properties of both the aquifer and the aquitard. Characterization of aquifer and aquitard properties is then necessary for the proper modeling of groundwater flow and contaminant transport. Despite that, to our knowledge there has been no formal attempt to relate the parameters determined with conventional interpretation methods to the heterogeneous distribution of the aquifer-aquitard parameters.

[9] Specifically, we simulate pumping tests in synthetically generated aquifer-aquitard systems, considering separately the heterogeneity of the pumping aquifer and the confining aquitard. Estimates of the apparent hydraulic parameters are obtained by analyzing each pumping test independently for different distances from the pumping well and using common methods available in the literature and used extensively in computer codes developed for the analysis of pumping tests. By combining the estimates of the apparent parameters obtained with the different analysis methods, it is shown that information about the heterogeneity of the system may be inferred.

[10] It is important to note that the interpretation of pumping tests conducted in heterogeneous media can, in principle, be formulated as a geostatistical or inverse problem. Over the past two decades several inversion modeling tools have been developed [see, for example, *Rubin*, 2003]. Some authors have also proposed the use of hydrological tomography for the estimation of spatial distribution of the flow parameters [*Yeh and Liu*, 2000; *Zhu and Yeh*, 2005] However, the computational effort involved in these interpretation approaches can be intensive and quite complex because of the ill-posedness of the groundwater inverse problem. The goal in this paper is to explore the possibility of extending commonly used pumping tests analysis procedures to extract information about the underlying spatial variability of leaky aquifer systems.

2. Problem Statement

2.1. Definition of the Heterogeneous Aquifer-Aquitard System

[11] The vertical cross-section of the leaky aquifer system considered in this study is similar to that analyzed by *Hantush and Jacob* [1955]. The system consists of two

horizontally unbounded aquifers, separated by an aquitard. A fully penetrating pumping well of infinitesimal radius is placed in one aquifer and is fully isolated from the other. Before pumping, the system is assumed to be at equilibrium, with both aquifers and the aquitard having the same hydraulic head. The aquitard is assumed to have no storage capacity. Because of the large contrast in hydraulic conductivity values, the flow induced by the pumping is approximately vertical in the aquitard and horizontal in the aquifer. The mathematical equation describing flow in the pumped aquifer is:

$$\frac{\partial}{\partial x}\left(T\frac{\partial s}{\partial x}\right)+\frac{\partial}{\partial y}\left(T\frac{\partial s}{\partial y}\right)-Cs=S\frac{\partial s}{\partial t} \tag{1}$$

where $T(x, y)$ [L^2T^{-1}] is the transmissivity of the (pumped) aquifer; $C(x, y)$ [T^{-1}] is the aquitard conductance, equal to the vertical hydraulic conductivity of the aquitard divided by aquitard thickness; S [−] is the aquifer storativity; $s(x, y, t)$ [L] is the drawdown. The leakage factor B [L] is defined as

$$B=(T/C)^{1/2}. \tag{2}$$

[12] For a homogeneous aquifer-aquitard system the transient drawdown is given by [*Hantush and Jacob*, 1955]:

$$s(r,t)=\frac{Q}{4\pi T_0}W(u,r/B), \tag{3}$$

where r is the radial coordinate, with origin at the pumping well, $s(r, t)$ is the drawdown; Q is the constant pumping rate, T_0 is the spatially uniform transmissivity, and $W(u, r/B)$ is the leaky well function:

$$W(u,r/B)=\int_u^{\infty}\frac{1}{y}\exp\left(-y-\frac{r^2}{4B^2y}\right)dy \tag{4}$$

with $u = r^2 S_0/4tT_0$, and S_0 the spatially uniform storage coefficient.

[13] Because of leakage through the semi-confining layer, the drawdown reaches a steady-state asymptotically. The steady drawdown, s_m, is given by [*de Glee*, 1930]:

$$s_m=\frac{Q}{2\pi T_0}K_0(r/B), \tag{5}$$

where K_0 is the modified Bessel function of the second kind and of order 0.

2.2. Existing Parameter Interpretation Methods

[14] Two procedures are commonly used in the analysis of time-drawdown data in leaky aquifers: (1) the inflection-point method [*Hantush*, 1956], and (2) the curve-fitting method [*Walton*, 1962]. Both methods are based on the assumption of homogeneity of the system, plus a number of additional somewhat restrictive assumptions, such as no storage released from the aquitard and constant head in the unpumped aquifer. Subsequent to the original study by *Hantush and Jacob* [1955], a number of papers relaxed some of the assumptions listed above. For example, *Hantush* [1960] accounted for the storage of the confining layer. *Neuman and Witherspoon* [1969a, 1969b] provided a more general solution including drawdown in the overlying aquifer. *Moench* [1985] incorporated the effect of the extraction well diameter and well bore skin on the transient drawdown of leaky aquifers. Despite these improvements, the above-mentioned methods remain quite popular for the interpretation of pumping test data in leaky aquifers because of their relative simplicity.

[15] For completeness, the inflection-point method and the curve-fitting method are described here briefly. The inflection-point method [*Hantush*, 1956] uses analytically derived relationships of the drawdown versus log-time curve to estimate the flow parameters. In particular, it is found that the ratio between the drawdown, s_p, and its derivative at the inflection-point location, Δs_p, is function of the ratio r/B only [*Hantush*, 1956]:

$$2.3\frac{s_p}{\Delta s_p}=\exp(r/B)K_o(r/B). \tag{6}$$

[16] The actual time where the inflection point takes place, t_p, can be written in terms of the different system parameters:

$$t_p=\frac{rSB}{2T}. \tag{7}$$

[17] Once the leakage factor, B, is estimated, the transmissivity, T, and storativity, S, of the perturbed aquifer can be determined sequentially from equations (5) and (7).

[18] It is possible to demonstrate analytically that the steady-state drawdown, s_m, is twice the drawdown at the inflection point $s_m = 2s_p$. Hence, in real applications, there are two different ways of applying the Hantush inflection point method. First, from the steady-state drawdown (or its extrapolation if the time-drawdown record is not sufficiently long), one can estimate $s_p = 0.5s_m$. Then from the drawdown curve, we estimate T_p and Δs_p. Alternatively, one can locate the inflection-point as the point where the derivative of the drawdown vs log-time is maximum, and from that determine s_p and Δs_p. While in homogeneous media both methods would render the same parameter values, in real (heterogeneous) media this is not necessarily true. The two variants of the method will be compared and evaluated as part of the present study.

[19] The curve-fitting method [*Walton*, 1962] finds the representative hydraulic parameters by comparing the observed time-drawdown data on a log-log plot to a family of type curves developed based on the analytical solution given in equation (3). In this paper, the best-fit parameters were determined by a trial and error approach that minimizes the sum of squared differences between the simulated drawdown and the theoretical drawdown:

$$\sum_i\left[\bar{s}(r,t_i)-s(r,t_i)\right]^2 \tag{8}$$

where $\bar{s}(r, t_i)$ is the observed drawdown at distance r from the pumping well and time t_i and $s(r, t_i)$ is the theoretical drawdown derived from the type curves.

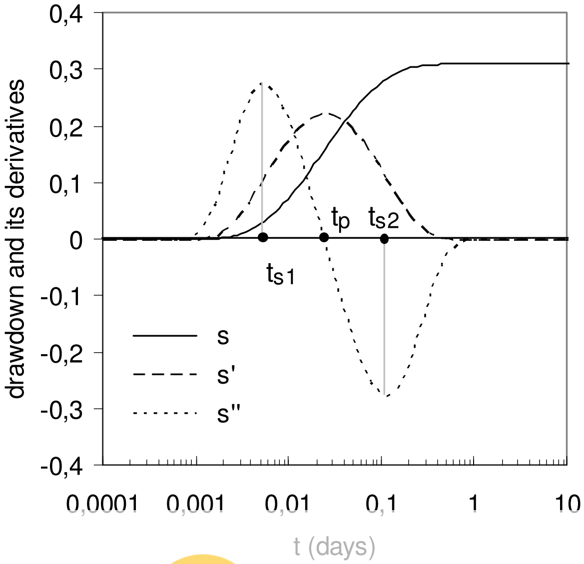


Figure 1. Drawdown and its first and second derivatives as a function of log time for Q = 2 m^3/d, T = 1 m^2/d, S = 0.0001, C = 0.001 d^{-1}, and r/B = 0.5 (adapted from *Trinchero et al.* [2008a]).

[20] Recently, a third method for the interpretation of pumping tests in leaky aquifers was developed, referred to as the double inflection-point (DIP) method [*Trinchero et al.*, 2008a]. The DIP method requires the estimation of the ratio $\tau = t_{s1}/2t_p$ or $\tau = t_{s2}/2t_p$ where t_p is the time corresponding to the inflection point (equation (7)) and t_{s1} and t_{s2} are the times corresponding to the two inflection-points of the first derivative of the drawdown with respect to the logarithm of time (Figure 1). Once τ is estimated, the leakage factor is computed directly from [*Trinchero et al.*, 2008a]:

$$B = \frac{\left(\tau^2 - 1/4\right)^2 r}{\tau(\tau^2 + 1/4)}. \tag{9}$$

[21] After the leakage factor, B, is estimated, the other flow parameters (T, S, and C) are calculated from equations (5), (7) and (2), respectively. Because τ can be based either on t_{s1} or t_{s2}, two sets of parameters are estimated with the DIP method.

[22] The above methods provide the exact solution for the hydraulic parameters (T, S, B) in a homogenous system, provided the drawdown data are noise-free. On the other hand, each method provides a different set of parameters in real (heterogeneous) systems, since each particular method gives more weight to different portions of the time-drawdown data. The emphasis that each method gives to the different portions of the drawdown curve is discussed further by *Trinchero et al.* [2008a] and section 3.3 of this paper. One of the main conclusions of Trinchero et al. was that by comparing the estimates of B provided by the inflection-point method [*Hantush*, 1956] and the DIP method, it is possible to infer information about the local transmissivity in the vicinity of the pumping well.

[23] In this study Monte Carlo simulations are used to assess what parameter values are given by the different methods and whether it is possible to relate these parameters to some characteristic values of the heterogeneous system.

2.3. Numerical Setup

[24] We model the natural logarithms of the transmissivity and aquitard hydraulic conductance as two independent multivariate Gaussian SRF's with stationary exponential semivariograms. Two sets of simulations are presented. In the first one the transmissivity of the pumped aquifer was assumed spatially variable while the aquitard conductance is assumed uniform. In the second set, the aquifer was assumed to be homogeneous and the aquitard heterogeneous. This allows us to evaluate the impact of the heterogeneity of the aquifer and the confining layer on the estimated parameters independently.

[25] For each set of parameters, 1000 realizations of the spatially variable parameter (aquifer transmissivity or aquitard conductance) were generated using the turning bands method [*Mantoglou and Wilson*, 1982]. The storativity of the pumped aquifer was assumed to be uniform equal to 0.0001. The confining aquifer was assumed to have no storage, which is consistent with the leaky aquifer system analyzed by *Hantush and Jacob* [1955]. The pumping well is located at the center of a square aquifer, 481 m on each side. The medium was discretized using square cells of size 1 × 1 m^2. At steady state, the equivalent well radius is approximately 0.2 times the cell size [*Desbarats*, 1992]. A prescribed head condition was imposed at the outer boundary, and the extraction well is treated as a prescribed sink term with steady rate of 2 m^3/d. Drawdowns were simulated using the finite difference model MODFLOW [*Harbaugh et al.*, 2000]. The test duration was 2 days, and a variable time step was used in the simulations, starting with 1 s, and gradually increasing it as the test progressed.

[26] Inspection of the results showed that steady state was reached at the end of the 2-day period for all cases considered. Some simulations were repeated with a no-flow condition at the outer boundary and the simulated drawdown data were identical, indicating that the outer boundary had no impact on the simulated drawdowns. The numerical setup was also tested by simulating the pumping test in a homogeneous leaky aquifer system and comparing the drawdown data to equation (3).

3. Results

3.1. Impact of Aquifer Heterogeneity

[27] The first set of results corresponds to the case with spatially uniform aquitard conductance (C_o = 0.001 d^{-1}) and spatially variable aquifer transmissivity (with geometric mean, T_g = 1 m^2/d, and ln T integral scale: I = 8 m, and variance: σ^2 = 1). Figure 2 compares the aquifer transmissivity estimates (normalized by T_g) using the inflection-point and the curve-fitting methods at different distances from the pumping well. Each point on the plots corresponds to one of the 1000 Monte Carlo realizations. Figure 2 also shows the correlation (Corr) and the mean absolute difference (MAD) between the two sets of estimates as a function of distance. From the two options of the inflection-point method described in section 2.2, we selected to use the slope and drawdown at the point corresponding to half the steady-state drawdown.

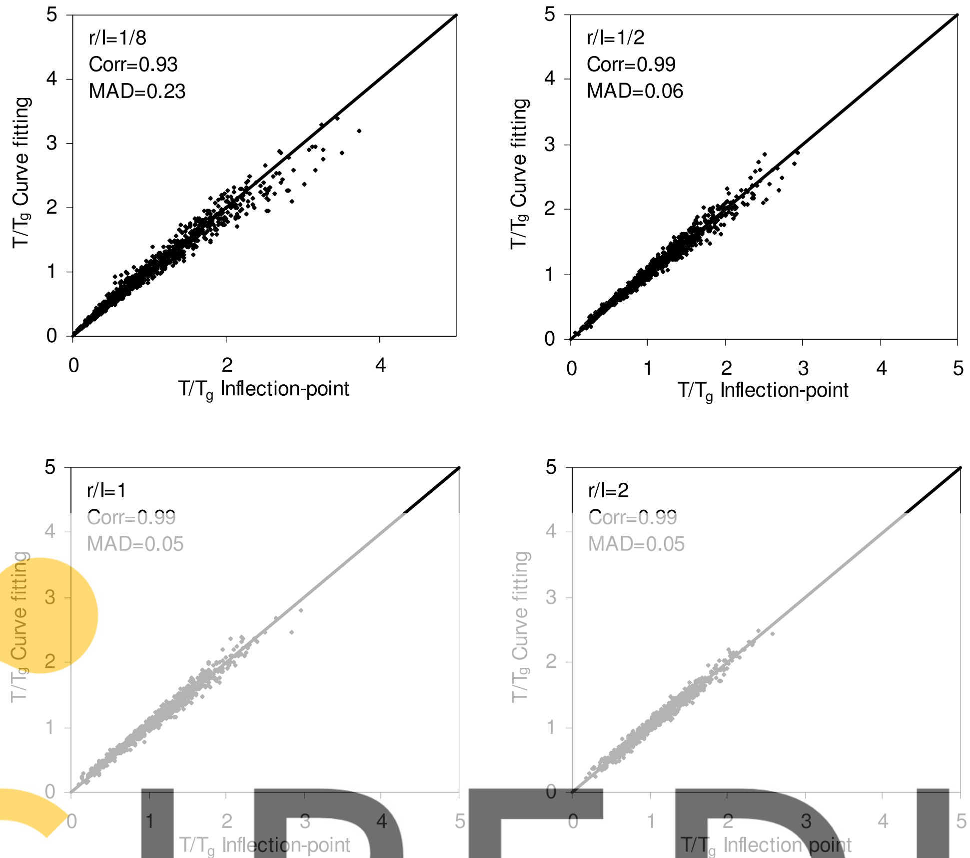


Figure 2. Normalized transmissivity estimates using the inflection-point and the curve-fitting methods for different distances from the pumping well (heterogeneous aquifer with $T_g = 1$ m^2 d^{-1}, $I = 8$ m, and $\sigma^2 = 1$ and a uniform aquitard with $C_o = 0.001$ d^{-1}).

[28] From Figure 2 we observe that overall the transmissivity estimates obtained with the two methods tend to spread around the 1:1 line. The differences between the values estimated with the two methods come from each model weighing differently the T values within the domain. For larger distances from the pumping well, the estimates with the two methods tend to converge, since the larger aquifer volume involved in the weighting process averages out the effect of heterogeneities. The convergence in the estimates is also reflected in the increasing Corr toward 1 and decreasing MAD toward zero with distance.

[29] There is a significant difference between the behavior of leaky and confined aquifers during pumping tests. In the latter, it was shown analytically by *Sanchez-Vila et al.* [1999b] that eventually all the estimated transmissivity values converge to a single T value which is close to the geometric average of the random function T(x, y). On the other hand, for pumping tests conducted in leaky aquifers, each realization yields potentially different estimates. The reason is that in confined aquifers steady-state conditions are never reached, and so, all points in the domain are eventually affected by pumping. This is not the case in leaky aquifers, where steady-state conditions are eventually reached, and the drawdown is null everywhere except within a finite volume around the pumping well. Thus the weighted average of the local T values can be different for each individual realization and, in particular, different from the overall mean value, T_g.

[30] Figure 3 displays scatter plots of the normalized aquitard conductance estimates from both the inflection-point and the curve-fitting methods. For small distances from the pumping well, the estimates with the two methods may differ significantly and the scatter of the estimated aquitard conductance values is greater than the scatter of the aquifer transmissivity values. For large distances the estimates become independent of the interpretation method. However, for many simulations the estimated values can be significantly different from the actual value used in the drawdown simulation (i.e., $C/C_o \neq 1$). These observations are also confirmed by the values of Corr and MAD which approach one and zero, respectively with increase in distance. A similar behavior is observed for the estimated storativity (The results are not presented here for brevity).

[31] The main low-order statistics of the flow parameters estimates are compiled in Table 1. The mean of the T estimates is found to be between the geometric and arithmetic mean of the transmissivity (in this case $T_a/T_g = 1.65$). *Copty et al.* [2006] observed a similar behavior in the equivalent transmissivity for steady-state radially convergent flow in leaky aquifers and associated this effect to each realization sampling only a portion of the domain centered around the pumping well, thus forcing the expected mean toward the arithmetic average. Table 1 also shows a slight increasing trend in the mean of estimated transmissivity with distance. This observation is discussed later.

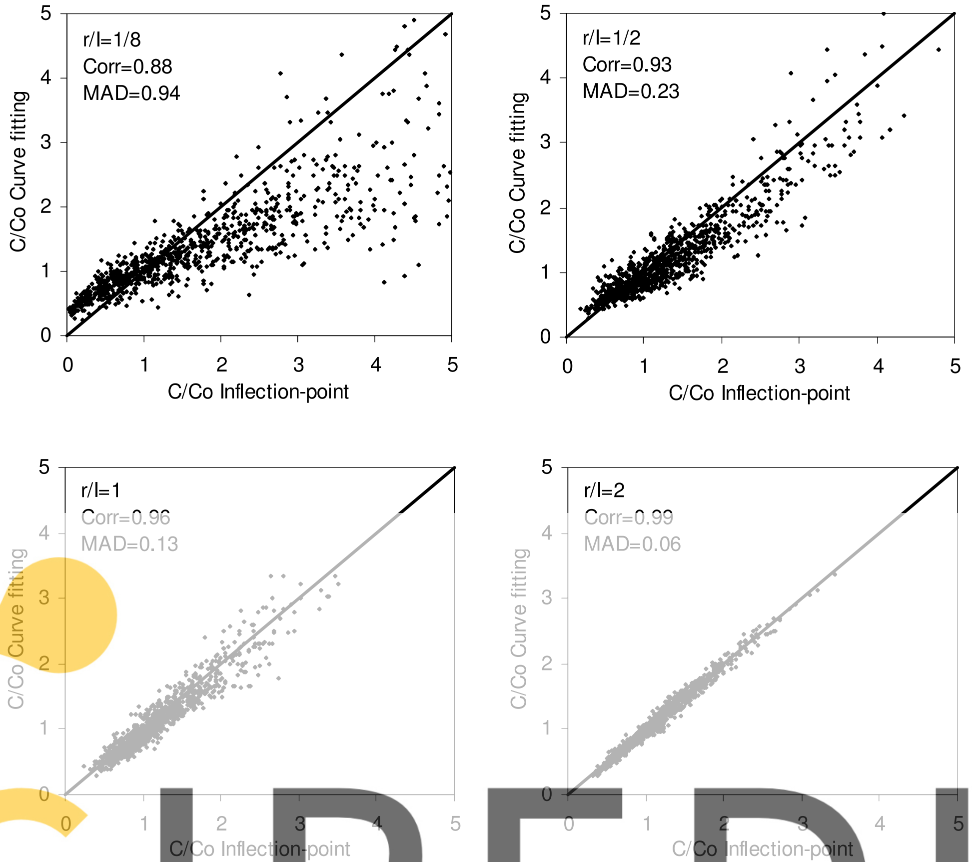


Figure 3. Normalized aquitard conductance estimated using the inflection-point and the curve-fitting methods for different distances from the pumping well (heterogeneous aquifer with T_g = 1 m^2 d^{-1}, I = 8 m, and σ^2 = 1 and a uniform aquitard with C_o = 0.001 d^{-1}).

[32] The deviation of the estimated T values from the actual distributions causes a bias in the storativity and aquitard conductance estimates, which in the mean are slightly overestimated. The reason comes from the interpretation methods, all of them leading to estimates that are not independent, but correlated. Thus errors in the estimation of one parameter directly translate to errors in the remaining parameters. All methods have a large uncertainty, as measured by the standard deviation (Table 1). For large distances the variances decrease and the estimates become more consistent.

[33] The probability density function (pdf) of the T/T_g estimates from the inflection-point method (depicted in Figure 2) is shown in Figure 4. The pdf of T/T_g is shown for different r/I values. For comparison, the (lognormal) distribution of the transmissivity values used in the generation of the heterogeneous T field is also shown. It is observed that all distributions are asymmetric with positive skewness. For observation points very close to the pumping well, the pdf of T/T_g is close to the log-normal distribution of the transmissivity field used in the pumping test simulation. As the value of r/I increases, the variance of T/T_g decreases. However, even for large values of r/I, the variance of T/T_g does not approach zero and, hence, the estimated transmissivity may differ from the geometric mean. As noted above, this is a significant difference in

Table 1. Expected Value and Standard Deviation (Shown in Parenthesis) of the Flow Parameters Function of Distance From the Well—Case of Spatially Variable Aquifer

Parameter	Interpretation Method	Distance From the Well r/I					
		1/8	1/2	1	2	4	8
Normalized transmissivity	Inflection point	1.06 (0.66)	1.07 (0.58)	1.07 (0.46)	1.07 (0.38)	1.11 (0.32)	1.24 (0.32)
	Curve fitting	1.07 (0.61)	1.08 (0.52)	1.08 (0.45)	1.08 (0.38)	1.12 (0.32)	1.24 (0.32)
Normalized leakage factor	Inflection point	1.03 (1.01)	1.01 (0.49)	1.01 (0.40)	1.03 (0.33)	1.06 (0.26)	1.09 (0.19)
	Curve fitting	0.93 (0.49)	1.03 (0.43)	1.05 (0.40)	1.06 (0.35)	1.07 (0.27)	1.09 (0.20)
Normalized conductance	Inflection point	2.48 (2.76)	1.34 (0.76)	1.20 (0.54)	1.13 (0.48)	1.07 (0.39)	1.08 (0.30)
	Curve fitting	1.74 (1.64)	1.22 (0.66)	1.15 (0.53)	1.11 (0.49)	1.08 (0.39)	1.08 (0.29)
Normalized storativity	Inflection point	1.98 (2.88)	1.25 (0.47)	1.19 (0.37)	1.16 (0.42)	1.14 (0.39)	1.19 (0.34)
	Curve fitting	1.61 (1.68)	1.18 (0.33)	1.17 (0.37)	1.16 (0.42)	1.14 (0.39)	1.18 (0.33)

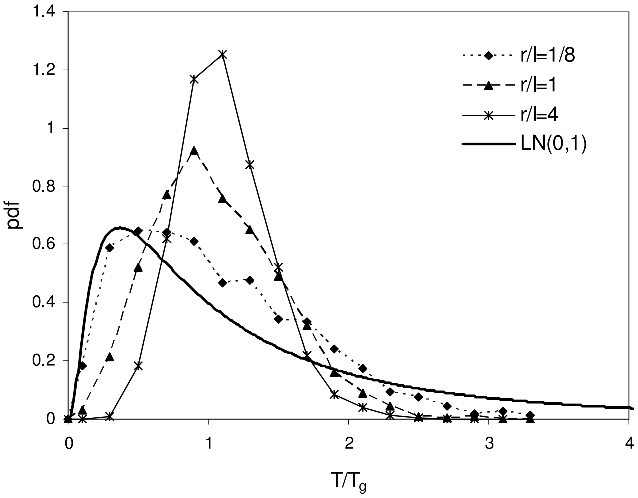


Figure 4. Probability density function of T/T_g estimated using the inflection-point method at different distances from the well. The lognormal ln(0, 1) distribution is also shown (heterogeneous aquifer with T_g = 1 m^2 d^{-1}, I = 8 m, and σ^2 = 1 and a uniform aquitard with C_o = 0.001 d^{-1}).

the behavior of leaky and confined aquifers where, in the latter case, the cone of depression continues to grow with time and a much larger aquifer volume contributes flow toward the pumping well.

[34] The distribution of the T/T_g estimates is dependent on the parameters of the aquifer-aquitard system, namely the values of C, I, and the σ^2. Figure 5 presents the pdf value of the T/T_g estimates for different sets of parameters and for r/I = 1. With increase in the σ^2, the variability of the transmissivity field increases and consequently, the pdf of the estimated T displays larger variance and skewness. As I or C increases, the aquifer volume sampled by the pumping test relative to the characteristic length of the transmissivity distribution diminishes. This yields estimates that are influenced more by the transmissivity near the pumping well and, hence, the pdf becomes closer to the log-normal distribution used in the generation of the T field.

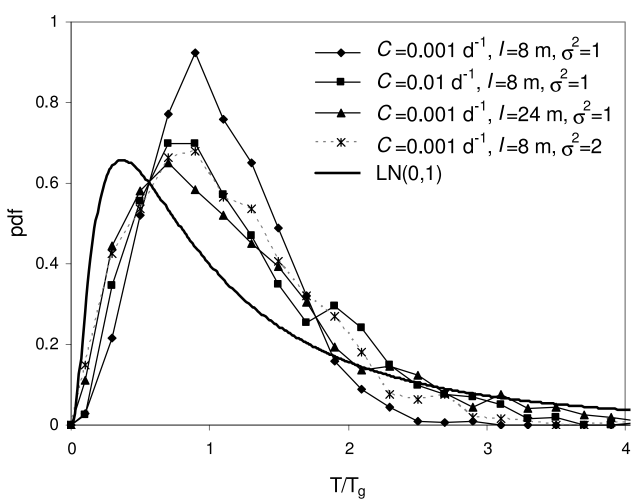


Figure 5. Sensitivity of the T/T_g pdf estimated using the inflection-point method to the conductance and log-transmissivity integral scale and variance. The lognormal distribution ln(0, 1) is also shown.

3.2. Spatial Variability of the Estimated Flow Parameters

[35] Figures 6a and 6b show the estimates of leakage factor and transmissivity (normalized by $B_g = (T_g/C_o)^{1/2}$ and

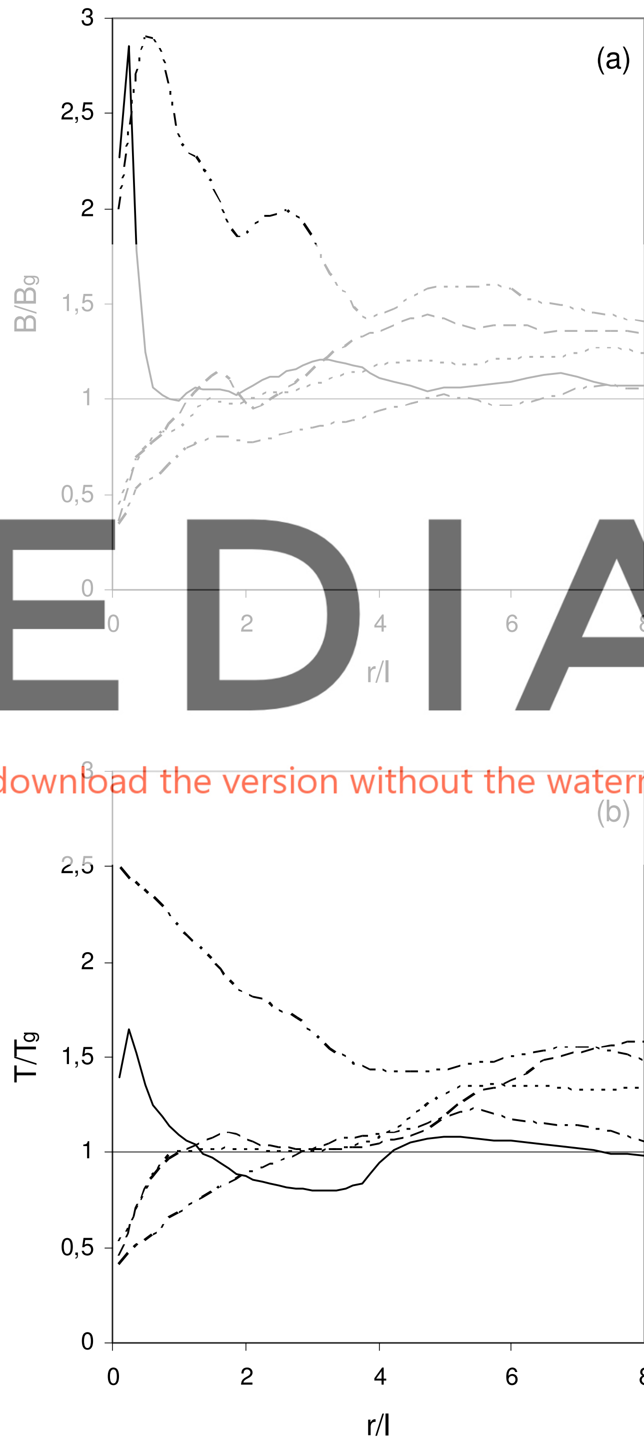
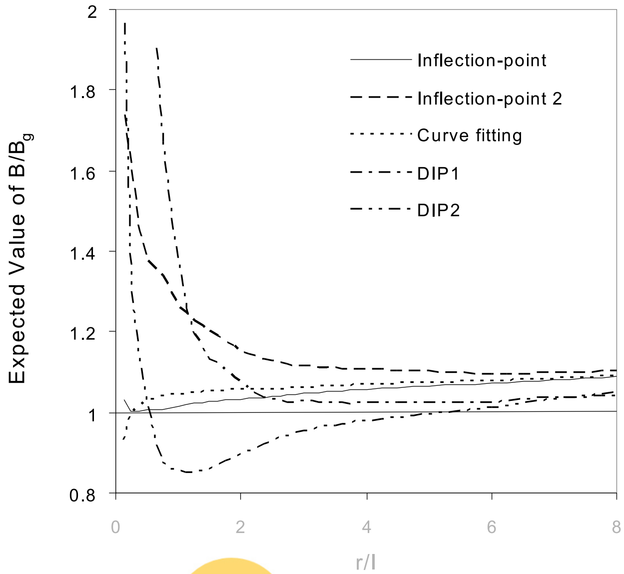


Figure 6. (a) Leakage factor normalized by $B_g = (T_g/C_o)^{1/2}$ and (b) transmissivity normalized by T_g estimated using the inflection-point method for randomly selected simulations as a function of distance from the well.



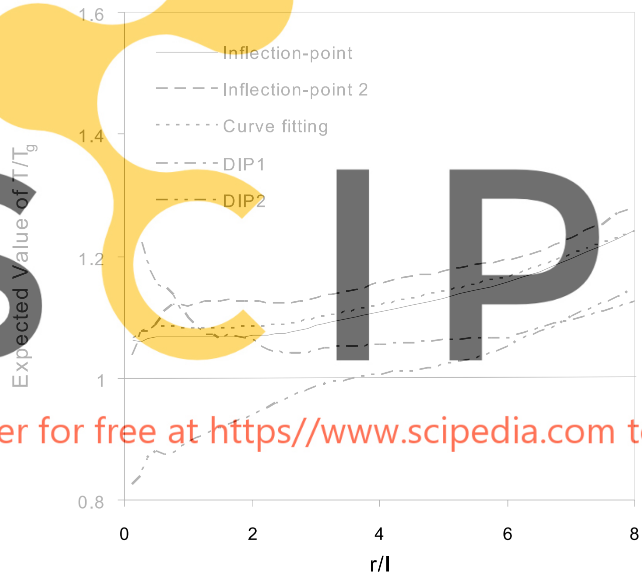


Figure 7. Expected value of the (a) leakage factor normalized by $B_g = (T_g/C_o)^{1/2}$ and (b) transmissivity normalized by T_g as a function of distance from the well (heterogeneous aquifer with $T_g = 1\ \mathrm{m^2\ d^{-1}}$, $I = 8$ m, and $\sigma^2 = 1$ and a uniform aquitard with $C_o = 0.001\ \mathrm{d^{-1}}$).

T_g respectively), for selected simulations and as a function of distance of the observation well from the pumping well. Only the estimates which were obtained with the inflection-point method are shown (those with the curve-fitting method are qualitatively similar). The resulting estimates are strongly dependent on the distance of the observation point from the pumping well and even for large distances ($r/I = 8$) generally display a noticeable variation for every individual realization. This behavior is very different to the confined case, where for large distances all the values of estimated transmissivities tend to converge to a single value that is close to the geometric mean of the T field [*Meier et al.*, 1998; *Sanchez-Vila et al.*, 1999a, 1999b].

[36] These results can potentially have significant implications in real applications where flow parameters estimated from pumping tests are used in geostatistical studies and for defining input parameters of groundwater flow models. Specifically, pumping tests conducted in the same formation would yield different flow parameters depending on the leakage into the pumped aquifer and on the location of the observation point relative to the pumping well.

[37] The spatial variability of the flow parameters is also observed in the ensemble mean of the flow parameters, average over the 1000 Monte Carlo simulations. Figures 7a and 7b show a comparison of the mean estimates of the leakage factor and transmissivity obtained from the different interpretation methods as a function of distance. Close to the pumping well, the mean leakage factors obtained with the different interpretation methods show large variations. With increasing distance, the mean estimates converge to a value slightly larger than the geometric mean, B_g. This effect can be explained by physical considerations. For heterogeneous leaky aquifers, the steady-state drawdown can be expressed as an extension of equation (5):

$$s_m = \frac{Q}{2\pi T_m} K_0(r/B_m), \tag{10}$$

where s_m is the steady-state drawdown and T_m and B_m are the apparent transmissivity and leakage factor, respectively, at steady state. Similarly, the drawdown derivative at the inflection-point can be expressed using the analytical solution of *Hantush* [1956] as:

$$\Delta s_p = \frac{2.3Q}{4\pi T_p} \exp(-r/B_p), \tag{11}$$

where Δs_p is the slope of the drawdown curve at the inflection-point and T_p and B_p are the apparent transmissivity and leakage factor, respectively, at the inflection point. Taking the ratio of equations (10) and (11) yields a somewhat modified version of equation (6):

$$2.3\frac{s_m/2}{\Delta s_p} = \frac{T_p}{T_m} \exp(r/B_p) K_o(r/B_m). \tag{12}$$

[38] Whereas T_m and B_m result from drawdown data at late times and, hence, are influenced by a larger aquifer volume, T_p and B_p are estimated from data at much earlier times. For observation points located close to the well, T_p and B_p may significantly differ from T_m and B_m. As the distance from the pumping well increases, the drawdown curve involves a larger portion of the aquifer, with the ratios T_p/T_m and B_p/B_m moving progressively closer to one. Both ratios however do not necessarily converge to 1 because of the limited aquifer volume that influences pumping tests in leaky aquifer. Thus, for large distances the response of the heterogeneous system approaches somewhat that of an equivalent homogeneous system. A similar analysis could also be applied to the DIP method since it also combines observations of the drawdown curve and its derivatives at different times.

[39] Figure 7 also shows the expected value of the normalized B and T estimates corresponding to the two variants of the inflection-point method, namely (1) from the slope at the point corresponding to half the steady-state drawdown (denoted as Inflection-point in the figure), and

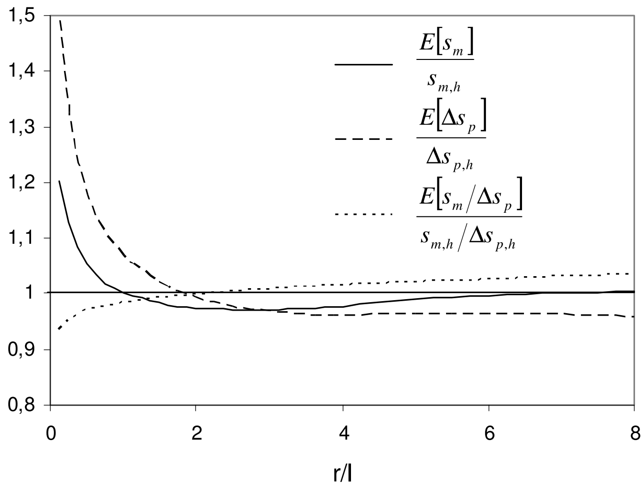


Figure 8. Comparison of the mean steady-state drawdown and drawdown slope at the inflection-point for a case of spatially variable transmissivity with that of the homogeneous aquifer with transmissivity T_g.

(2) from the maximum slope of the drawdown curve (Inflection-point 2 in the figure). In the latter case, the corresponding expression relating the ratio of drawdown to drawdown slope at the inflection-point to the leakage factor is:

$$2.3\frac{s_p}{\Delta s_p} = \exp(r/B_p)K_o(r/B_p). \tag{13}$$

[40] As evidenced by equation (12), the leakage factor estimated with the first variant of the inflection-point method actually depends on the apparent parameters at both intermediate and late (steady) time. On the other hand, from equation (13) we see that estimates obtained with the Inflection-point 2 are influenced by the apparent hydraulic parameters at intermediate times only. Hence the former interpretation (equation (12)) of the inflection-point method is relatively less dependent on the local conditions around the pumping well than Inflection-point 2.

[41] Another point observed in Figure 7 is the increasing trend in both the estimated leakage factor and transmissivity with distance obtained regardless of the interpretation method (except for low values of r/I). The explanation behind this trend for the inflection-point method is as follows. Figure 8 shows the expected values of s_m, Δs_p, and $s_m/\Delta s_p$. All three quantities are normalized by their counterparts $s_{m,h}$, $\Delta s_{p,h}$, and $s_{m,h}/\Delta s_{p,h}$ corresponding to the homogeneous aquifer with transmissivity equal to T_g. For large distances both s_m and Δs_p are underestimated, but s_m converges faster to its homogeneous counterpart, so that the normalized ratio $s_m/\Delta s_p$ is greater than 1 and increases monotonously in the range of distances explored. From equation (6), the direct consequence of overestimating $s_m/\Delta s_p$ is to overestimate B, and, subsequently, overestimate T (Figure 7b).

[42] A similar increasing trend in the estimated transmissivity of the pumped aquifer with distance between the observation and pumping wells was also observed by *Neuman and Witherspoon* [1969b]. Their analysis however was for the case of homogeneous media, with the apparent increase in the estimated transmissivity with distance resulting when the storage of the aquitard is neglected. As noted above, the apparent increase in our case is due to the heterogeneity of the aquifer and the fact that data from different times are used in the data interpretation.

3.3. Identification of the Local Transmissivity at the Pumping Well

[43] As indicated by *Trinchero et al.* [2008a], the DIP method can potentially be used to infer additional information on the local contrast of the transmissivity in the vicinity of the pumping well relative to the transmissivity spatial mean. To illustrate this, the DIP method and inflection-point method are jointly applied to the Monte Carlo simulations. Figures 9 and 10 compare the transmissivity estimates obtained from the inflection-point method with the two estimates obtained from the DIP method (DIP1 and

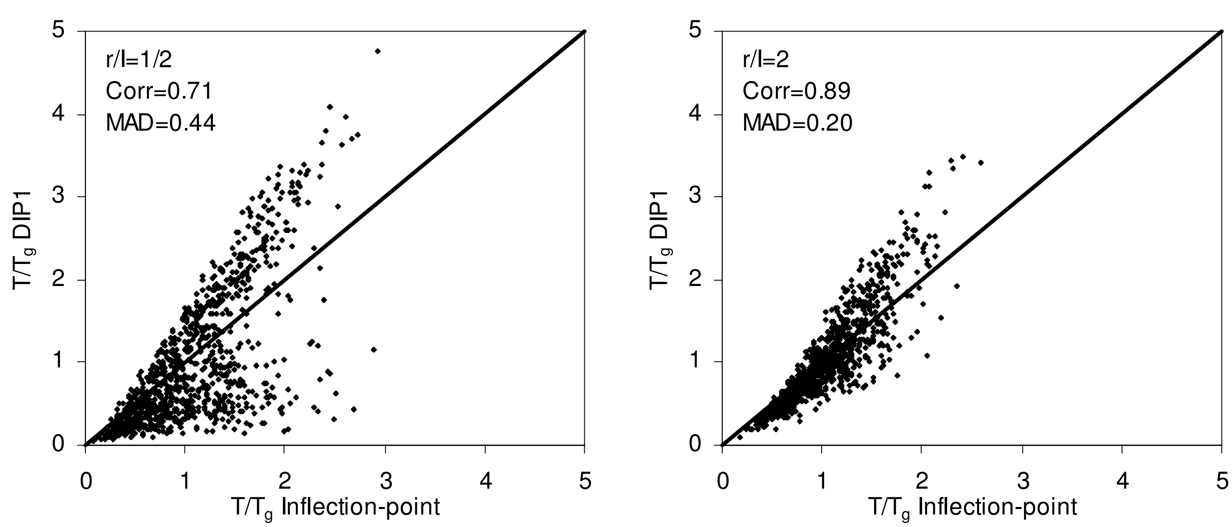


Figure 9. Normalized transmissivity estimated using the inflection-point and the DIP1 (positive peak) methods for $r/I = 1/2$ and $r/I = 2$ (heterogeneous aquifer with $T_g = 1\ \mathrm{m^2\ d^{-1}}$, $I = 8$ m, and $\sigma^2 = 1$ and a uniform aquitard with $C_o = 0.001\ \mathrm{d^{-1}}$).

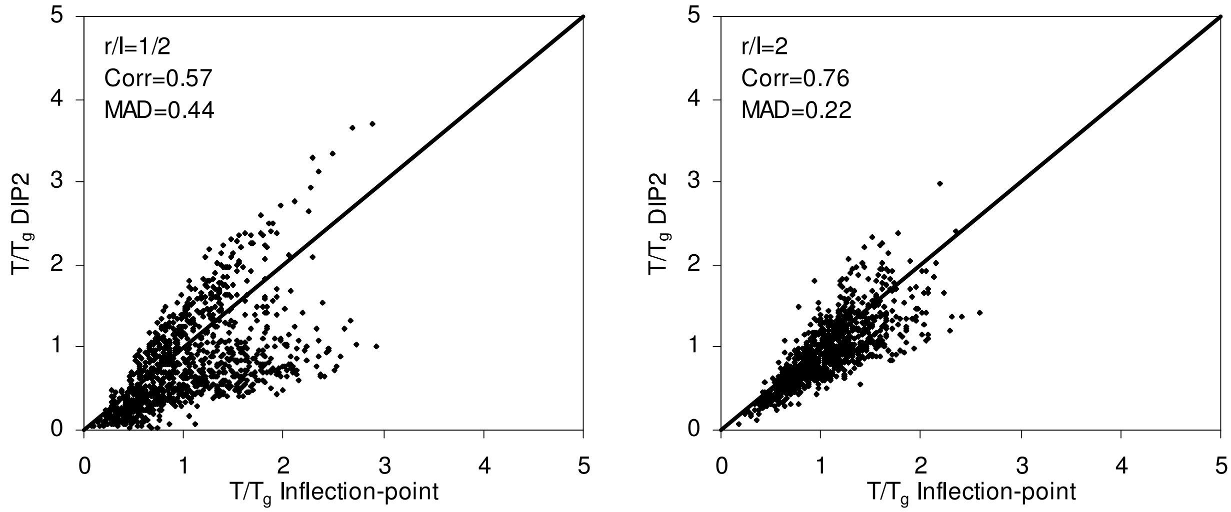


Figure 10. Normalized transmissivity estimated using the inflection-point and the DIP2 (negative peak) method for r/I = 1/2 and r/I = 2 (heterogeneous aquifer with T_g = 1 m^2 d^{-1}, I = 8 m, and σ^2 = 1 and a uniform aquitard with C_o = 0.001 d^{-1}).

DIP2), respectively, for two different distances from the pumping well (r/I = 1/2 and r/I = 2).

[44] Although the agreement among the different estimates improves with increase in distance from the pumping well, large differences are observed for some realizations. This is a result of each method giving emphasis to different portions of the time-drawdown data. The DIP1 method uses early time-drawdown data, and thus it tends to provide estimates that are representative of a very small volume around the pumping well. Therefore it tends to yield T estimates closer to the transmissivity near the pumping well. On the other hand, the DIP2 method places more emphasis on the late portion of the drawdown, providing T values that are representative of large aquifer volumes. As a consequence, the geometric average of the estimates from the two DIP methods which is presented in Figure 11 agrees better with the inflection-point estimates than each method individually. This can also be seen in the values of Corr and MAD shown on Figures 9–11.

[45] From the estimates obtained with the inflection-point (Hantush), DIP1 and DIP2 methods it is possible to infer information about the local transmissivity at the pumping well. For this purpose three correlation functions are defined:

[46] 1. $C(T_w, T_{D1}\text{-}T_H)$ is a measure of the correlation of the transmissivity at the pumping well to the difference between the inflection-point and DIP1 estimates.

[47] 2. $C(T_w, T_H\text{-}T_{D2})$ is a measure of the correlation of the transmissivity at the pumping well to the difference between the inflection-point and DIP2 estimates.

[48] 3. $C(T_w, T_{D1}\text{-}T_{D2})$ is a measure of the correlation of the transmissivity at the pumping well to the difference between the DIP1 and DIP2 estimates.

[49] The correlation coefficients were computed using the parameter values estimated from the time-drawdown data of individual simulations for different distances from the pumping well, and then by averaging over the 1000 Monte Carlo simulations. Because the estimates T_{D1}, T_H, and T_{D2} are influenced by progressively larger aquifer volumes,

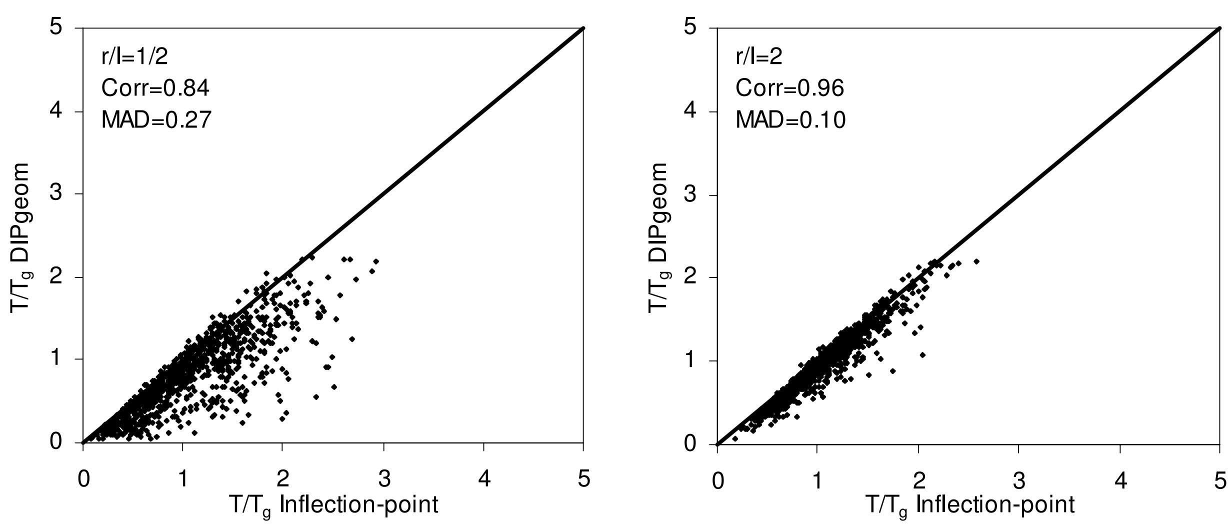


Figure 11. Normalized transmissivity estimated using the inflection-point method and the geometric mean of the two DIP estimates for r/I = 1/2 and r/I = 2 (heterogeneous aquifer with T_g = 1 m^2 d^{-1}, I = 8 m, and σ^2 = 1 and a uniform aquitard with C_o = 0.001 d^{-1}).

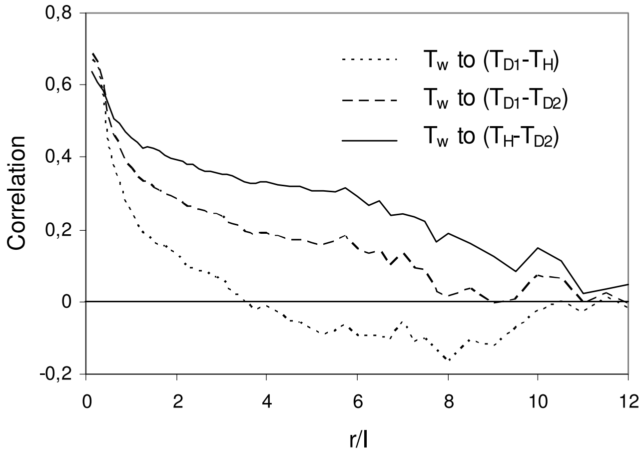


Figure 12. Correlation of the difference in the estimated transmissivity values obtained from different methods to the transmissivity at the well (heterogeneous aquifer with $T_{\rm g}$ = 1 m^2 d^{-1}, I = 8 m, and σ^2 = 1 and a uniform aquitard with $C_{\rm o}$ = 0.001 d^{-1}).

positive values of the three correlations T_{D1}-T_H, T_H-T_{D2} or T_{D1}-T_{D2} with T_w are indicative of a local T_w larger than the aquifer T spatial mean and vice versa. The three correlation functions are shown in Figure 12 as a function of distance from the pumping well. The correlation functions tend to vanish with distance, indicating that far from the well all the estimates are independent of the local T value at the pumping well. On the other hand, observation points located at $r < I$ would provide the greatest information about the local transmissivity.

[50] Finally, we note that from previously published work [e.g., *Desbarats*, 1992; *Copty et al.*, 2006], one would expect that the representative local transmissivity would be a time-dependent weighted average of the point T values surrounding the pumping well, with weights decreasing with distance. Since the correlations displayed in Figure 12 are in terms of the single transmissivity value at the pumping well, T_w, only partial correlations are observed.

3.4. Impact of Aquitard Heterogeneity

[51] In this section we consider the scenario where the aquifer is assumed to be homogeneous with transmissivity $T_{\rm o}$ = 1 m^2/d, while the aquitard conductance is assumed to be a multi-log-Gaussian SRF with a geometric mean $C_{\rm g}$ = 0.001 1/d. The variance and integral scale of the natural log transform of the conductance, ln C, are 1 and 8 m, respectively. For comparison purposes, these are identical to what was used earlier for the case of homogeneous aquitard and spatially variable transmissivity.

[52] The normalized transmissivity and conductance estimates obtained from the inflection-point and the curve-fitting methods for different distances from the pumping well are presented in Figures 13 and 14, respectively. The values used for normalization are: $T_{\rm o}$ and $C_{\rm g}$ = 0.001 1/d. Table 2 provides the expected values and standard deviations of all the estimated flow parameters, including storativity and leakage factor, at different distances.

[53] In this scenario, both estimation methods yield very similar transmissivity and storativity estimates, as seen also in the values of Corr and MAD. Further, for short distances, normalized expected values of T and S are very close to 1 with a very small standard deviation (Table 2). For larger distances, although the estimates obtained with both methods are in good agreement with each other, they deviate in many instances from $T_{\rm o}$ (see Table 2 and Figure 13). This indicates that, as the distance between the observation point and pumping well increases, the spatial variability of the aquitard conductance is expected to have a larger influence on the estimation of the transmissivity.

[54] The expected value of the estimated conductance on the other hand is close to the arithmetic mean ($C_g \exp(\sigma^2/2)$ = 1.65 · 10^{-3} 1/d), which translates into a mean estimated leakage factor smaller than the geometric mean defined as $L_g = (T_0/C_{\rm g})^{1/2}$. The standard deviation tends to slightly decrease with distance (Table 2 and Figure 14).

[55] This behavior can be explained by looking at the vertical fluxes through the semiconfining layer of a homogeneous leaky aquifer system. We consider an observation well at distance $r_{\rm obs}$ from the pumping well. The time corresponding to the inflection point for this observation well is given by: $t_p = r_{\rm obs}SB/2T$. At this moment in time (time of inflection), the leaky well function at any distance r from the pumping well is W(u, r/B) where $u = \dfrac{r^2 S}{4 t_p T} = \dfrac{(r/B)^2}{2 r_{obs}/B}$. A plot of W($u$, r/B) as a function r/B at $T_{\rm p} = r_{\rm obs}/2T$ and for different values of $r_{\rm obs}/B$ is shown in Figure 15a. Integrating the leaky well function over the entire aquifer yields the ratio of the cumulative vertical flow though the aquitard to the pumping rate at the well, which is shown in Figure 15b. For small values of $r_{\rm obs}/B$, only a small fraction of the pumped water is leakage through the confining layer. As such, the pumping test is influenced by the local aquitard conductance only. That is, the pumping test is not influenced by the spatial variability of the aquitard conductance, explaining why at short distances the expected value of the estimated transmissivity shows no spread around its actual value (Figure 13a). As $r_{\rm obs}/B$ increases, the perturbed confining layer volume increases and the vertical flow constitutes a larger fraction of the pumping rate. For such conditions, the pumping test would yield a weighted spatial average of the aquitard conductance and as a result, the transmissivity estimate would deviate from the actual values.

4. Conclusions

[56] This paper examines the impact of heterogeneity of leaky aquifer systems on the flow parameters estimated with three different methods, two of them commonly used in real applications: the inflection-point [*Hantush*, 1956], the curve-fitting [*Walton*, 1962] methods, plus the recently developed double inflection-point method [*Trinchero et al.*, 2008a].

[57] We simulate two framework scenarios whereby the aquifer or aquitard are assumed homogeneous, while the other is defined as a multi-Gaussian SRF with given geostatistical parameters. For the case of spatially variable transmissivity and uniform aquitard conductance, the following observations can be made:

[58] 1. For observation points located relatively far from the well, all interpretation methods yield similar estimates of the transmissivity, storativity and aquitard conductance.

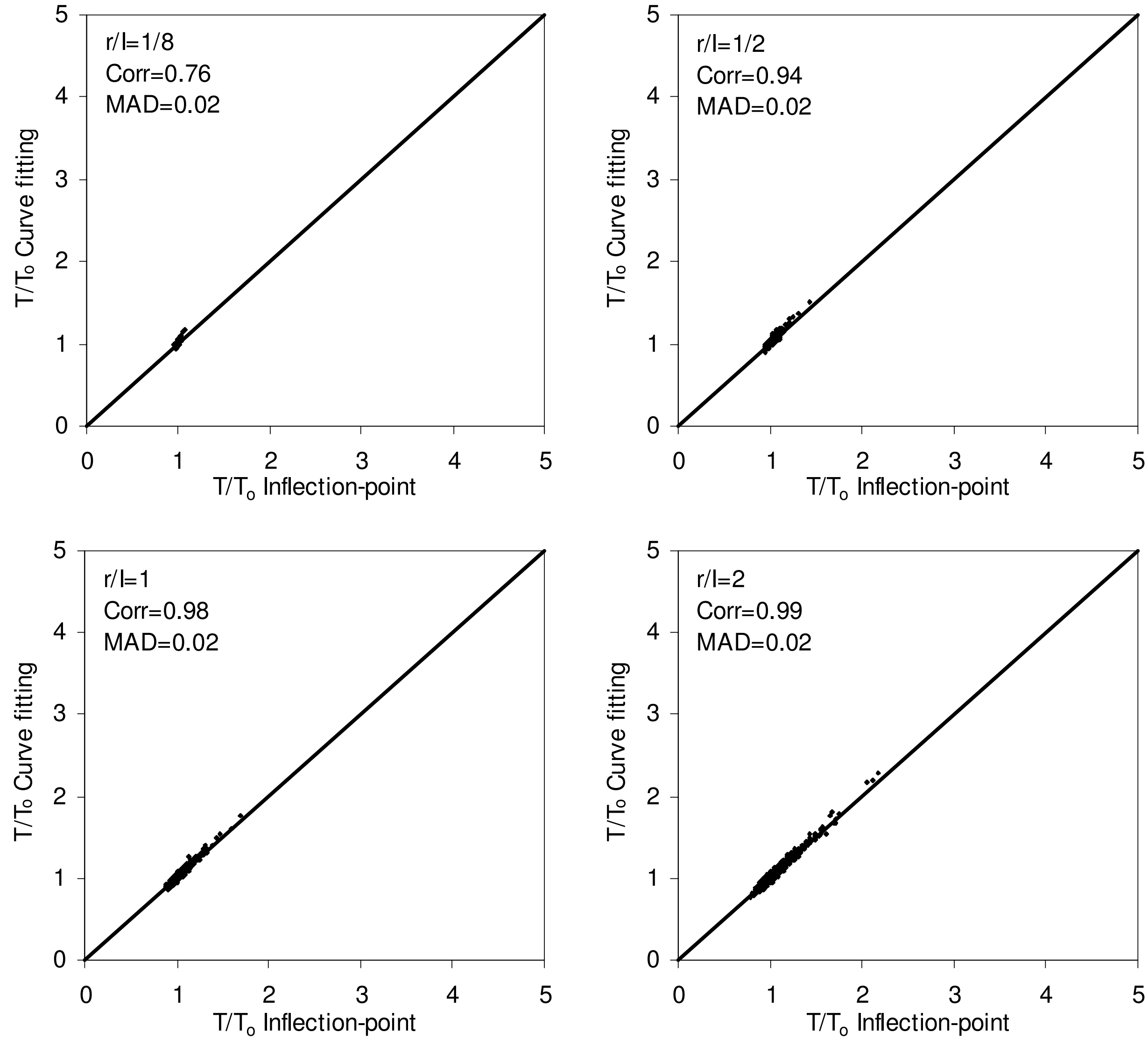


Figure 13. Normalized transmissivity estimated using the inflection-point and the curve-fitting methods for different distances from the pumping well (uniform aquifer with T_o = 1 m^{2}d^{-1} and spatially variable aquitard with C_g = 0.01 d^{-1}, I = 8 m, and σ^2 = 1).

[59] 2. The expected value of the transmissivity estimates depend on the geostatistical parameters of the T field, on the leakage factor, and the distance to the pumping well. For the Hantush inflection method and the Walton method the estimates are slightly larger than the geometric mean of the point T values.

[60] 3. The T estimates from the individual realization are dependent on the location of the observation point relative to the pumping well, in contrast to the case of confined aquifers where the estimated T is relatively insensitive to the observation well location and tends to be close to the geometric mean.

[61] 4. A slight increasing trend is observed in the expected value of the estimated leakage factor and transmissivity with distance from the pumping well.

[62] 5. Because the two DIP estimates rely on different portions of the time-drawdown data, they may differ from each other. The geometric mean of the two DIP estimates is generally in good agreement with the estimate obtained from the inflection-point method, particularly for distances greater than the integral scale of T. Moreover, differences in the estimates of T obtained with the inflection-point and DIP methods are correlated with the local value of T at the pumping well. This correlation tends to decrease with distance from the pumping well. Therefore, using the drawdown data at an observation point, located close to the well ($r < I$), it is possible to infer local contrasts in the transmissivity.

[63] The second scenario assumes a uniform aquifer transmissivity and a heterogeneous aquitard. On the basis of the results of these simulations we conclude:

[64] 6. The heterogeneity of the aquifer and aquitard influence the estimated hydraulic parameters in distinct manners. For the case of spatially variable aquitard, the agreement in the transmissivity and storativity estimates obtained with the inflection-point and the curve-fitting methods is very good near the pumping well. The estimation of the aquitard conductance shows an opposite trend.

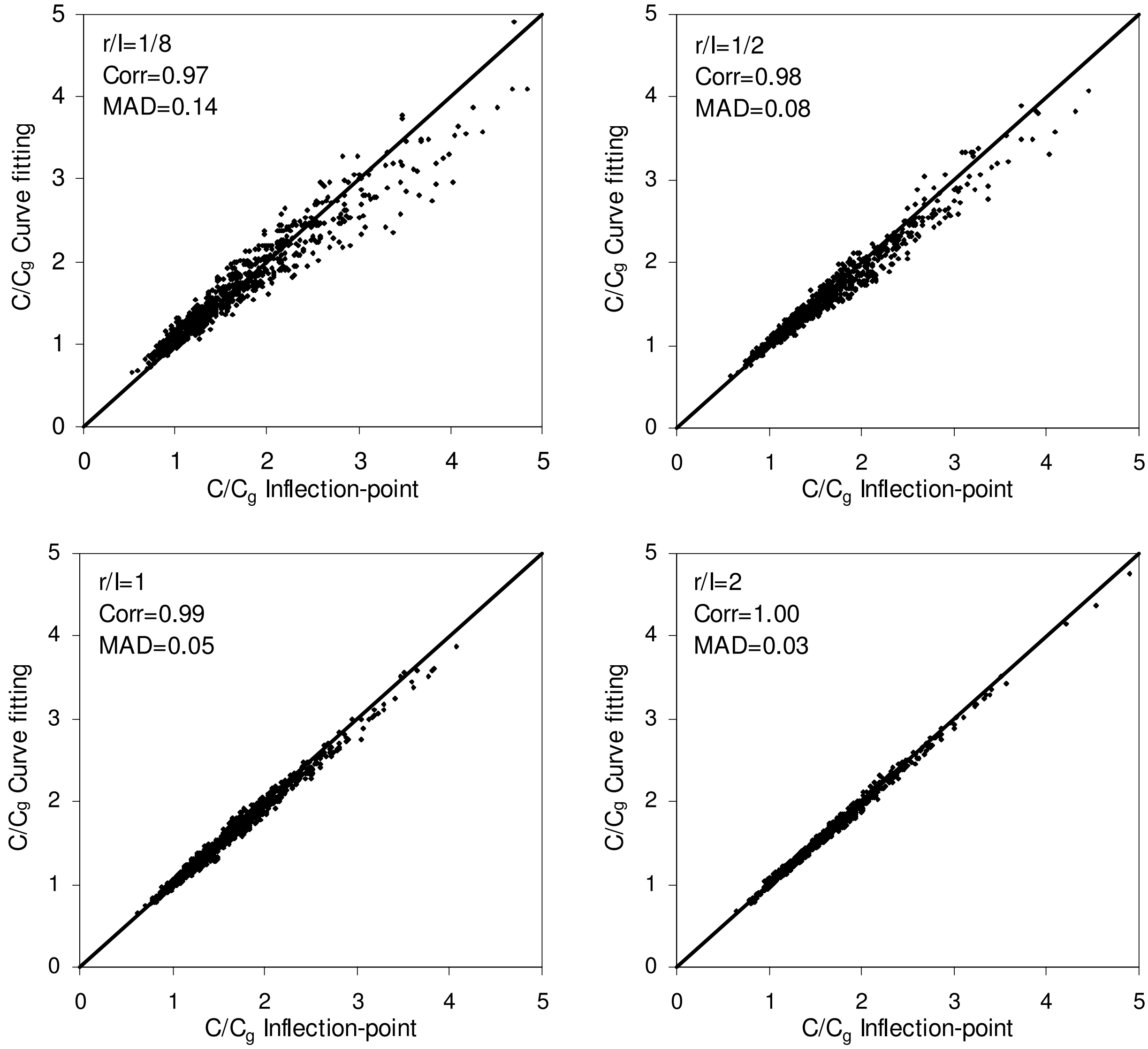


Figure 14. Normalized aquitard conductance estimated using the inflection-point and the curve-fitting methods for different distances from the pumping well (uniform aquifer with $T_o = 1$ m^{2}d^{-1} and spatially variable aquitard with $C_g = 0.01$ d^{-1}, $I = 8$ m, and $\sigma^2 = 1$).

With increasing distance, both T and S estimates exhibit larger variability.

[65] 7. The expected value of the aquitard conductance is close to the arithmetic mean of the C values, indicating that the drawdown is most sensitive to the local conductance at the pumping well, with the sensitivity rapidly decreasing with distance.

[66] Overall, this numerical exercise provides a framework to understand the implications of the assumption of homogeneity in the estimates obtained with the different

Table 2. Expected Value and Standard Deviation (Shown in Parenthesis) of the Flow Parameters Function of Distance From the Well—Case of Spatially Variable Aquitard

		Distance From the Well r/I					
Parameter	Interpretation Method	1/8	1/2	1	2	4	8
Normalized transmissivity	Inflection point	0.98 (0.01)	1.00 (0.05)	1.01 (0.08)	1.04 (0.15)	1.13 (0.27)	1.39 (0.44)
	Curve fitting	0.98 (0.03)	1.00 (0.06)	1.02 (0.09)	1.05 (0.16)	1.13 (0.26)	1.34 (0.36)
Normalized leakage factor	Inflection point	0.82 (0.15)	0.82 (0.13)	0.82 (0.11)	0.84 (0.10)	0.86 (0.08)	0.89 (0.06)
	Curve fitting	0.81 (0.13)	0.83 (0.12)	0.83 (0.11)	0.84 (0.09)	0.86 (0.08)	0.89 (0.06)
Normalized conductance	Inflection point	1.63 (0.74)	1.61 (0.61)	1.59 (0.54)	1.56 (0.50)	1.58 (0.52)	1.78 (0.67)
	Curve fitting	1.65 (0.65)	1.57 (0.55)	1.56 (0.52)	1.55 (0.49)	1.59 (0.53)	1.76 (0.63)
Normalized storativity	Inflection point	0.97 (0.01)	1.00 (0.01)	1.01 (0.02)	1.02 (0.07)	1.07 (0.17)	1.27 (0.33)
	Curve fitting	0.99 (0.02)	1.00 (0.02)	1.01 (0.03)	1.02 (0.08)	1.08 (0.17)	1.25 (0.28)

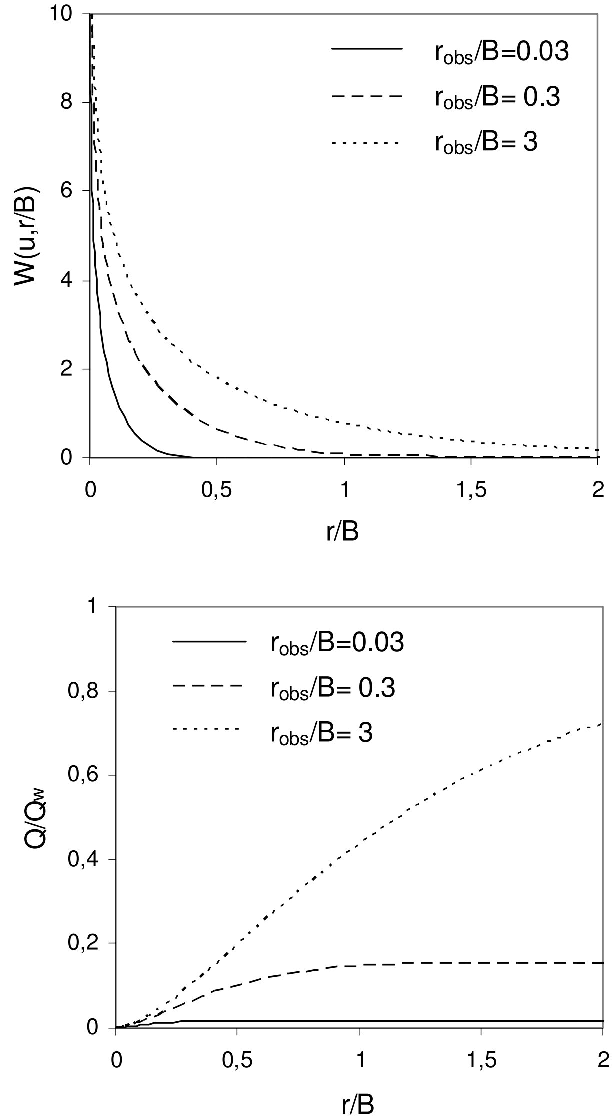


Figure 15. Leaky well function and vertical flow through the aquitard as a fraction of the pumping rate vs. r/B at the inflection point $t_p = r_{obs}SB/2T$ for different values of r_{obs}/B.

methods commonly used in the interpretation of pumping tests in aquifer-aquitard systems. Since each method give different emphasis to different portions of the drawdown curve and, consequently to different volumes of the aquifer-aquitard system, we conclude that using all analysis methods jointly may provide additional information (specifically, about contrasts in the local value of the transmissivity at the pumping well) than using each method independently.

[67] **Acknowledgments.** Nadim Copty acknowledges the financial support provided by the Scientific and Technological Council of Turkey (TUBITAK), project 104I130, and by the Bogazici University Research Fund (BAP), project 06Y104. Paolo Trinchero and Xavier Sanchez-Vila acknowledge support from CICyT (project PARATODO), the EU (project GABARDINE), and the Agència de Gestió d'Ajuts Universitaris i de Recerca of the Catalan Government. The constructive comments of Shlomo Neuman and Walter Illman are gratefully acknowledged.

References

Barker, J., and R. Herbert (1982), Pumping tests in patchy aquifers, *Ground Water*, *20*(2), 150–155.

Butler, J. J. (1988), Pumping tests in nonuniform aquifers: The radially symmetric case, *J. Hydrol.*, *101*(1–4), 15–30.

Butler, J. J., and W. Z. Liu (1993), Pumping tests in nonuniform aquifers: The radially asymmetric case, *Water Resour. Res.*, *29*(2), 259–269.

Cooper, H., and C. Jacob (1946), A generalized graphical method for evaluating formation constants and summarizing well-field history, *Trans., Am. Geophys. Union*, *27*(4), 526–534.

Copty, N. K., and A. N. Findikakis (2004a), Stochastic analysis of pumping test drawdown data in heterogeneous geologic formations, *J. Hydraul. Res.*, *42*, 59–67.

Copty, N. K., and A. N. Findikakis (2004b), Bayesian identification of the local transmissivity using time-drawdown data from pumping tests, *Water Resour. Res.*, *40*, W12408, doi:10.1029/2004WR003354.

Copty, N. K., M. S. Sarioglu, and A. N. Findikakis (2006), Equivalent transmissivity of heterogeneous leaky aquifers for steady state radial flow, *Water Resour. Res.*, *42*(4), W04416, doi:10.1029/2005WR004673.

Dagan, G. (1982), Analysis of flow through heterogeneous random aquifers. 2: Unsteady flow in confined formations, *Water Resour. Res.*, *18*(5), 1571–1585.

Dagan, G. (1989), *Flow and Transport in Porous Formations*, Springer, Berlin, Germany.

Dagan, G. (2001), Effective, equivalent and apparent properties of heterogeneous media, in *Mechanics for a New Millenium*, edited by H. Aref, and J. W. Phillips, pp. 473–485, Kluwer, Dordrecht, Netherlands.

de Glee, G. (1930), Over grondwaterstroomingen bij wateronttrekking door middle van putten, Ph.D. thesis, Delft Technische Hoogeschool, Delft.

Desbarats, A. J. (1992), Spatial averaging of transmissivity in heterogeneous fields with flow towards a well, *Water Resour. Res.*, *28*(3), 757–767.

Durlofsky, L. J. (2000), An approximate model for well productivity in heterogeneous porous media, *Math. Geol.*, *32*(4), 421–438.

Firmani, G., A. Fiori, and A. Bellin (2006), Three-dimensional numerical analysis of steady state pumping tests in heterogeneous confined aquifers, *Water Resour. Res.*, *42*, W03422, doi:10.1029/2005WR004382.

Gomez-Hernandez, J. J., and S. M. Gorelick (1989), Effective groundwater model parameter values: Influence of spatial variability of hydraulic conductivity, leakance, and recharge, *Water Resour. Res.*, *25*(3), 405–419.

Guadagnini, A., M. Riva, and S. P. And Neuman (2003), Three-dimensional steady state flow to a well in a randomly heterogeneous bounded aquifer, *Water Resour. Res.*, *39*(3), 1048, doi:10.1029/2002WR001443.

Hantush, M. (1956), Analysis of data from pumping tests in leaky aquifers, *Trans., Am. Geophys. Union*, *37*(6), 702–714.

Hantush, M. S. (1960), Modification of the theory of leaky aquifers, *J. Geophys. Res.*, *65*(5), 1634.

Hantush, M., and C. Jacob (1955), Non-steady radial flow in an infinite leaky aquifer, *Trans., Am. Geophys. Union*, *36*(1), 95–100.

Harbaugh, A., E. Banta, M. Hill, and M. McDonald (2000), Modflow-2000: The US Geological Survey modular ground-water model–user guide to modularization concepts and the ground-water flow process, *USGS Open-File Report 00-92*, p. 121 USGS, Reston, Va.

Indelman, P. (2003a), Transient well-type flows in heterogeneous formations, *Water Resour. Res.*, *39*(3), 1064, doi:10.1029/2002WR001407.

Indelman, P. (2003b), Transient pumping well flow in weakly heterogeneous formations, *Water Resour. Res.*, *39*(10), 1287, doi:10.1029/2003WR002036.

Indelman, P., and B. Abramovich (1994), Nonlocal properties of nonuniform averaged flows in heterogeneous media, *Water Resour. Res.*, *30*(12), 3385–3393.

Indelman, P., A. Fiori, and G. Dagan (1996), Steady flow towards wells in heterogeneous formations: Mean head and equivalent conductivity, *Water Resour. Res.*, *32*(7), 1975–1983.

Knight, J. H., and G. J. Kluitenberg (2005), Analytical solutions for sensitivity of well tests to variations in storativity and transmissivity, *Adv. Water Resour.*, *28*, 1057–1075.

Leven, C., and P. Dietrich (2006), What information can we get from pumping tests?—Comparing pumping test configurations using sensitivity coefficients, *J. Hydrol.*, *319*(1–4), 199–215.

Mantoglou, A., and J. l. Wilson (1982), The turning bands method for simulation of random-fields using line generation by a spectral method, *Water Resour. Res.*, *18*(5), 1379–1394.

Meier, P. M., J. Carrera, and X. Sanchez-Vila (1998), An evaluation of Jacob's method for the interpretation of pumping tests in heterogeneous formations, *Water Resour. Res.*, *34*(5), 1011–1025.

Moench, A. (1985), Transient flow to a large-diameter well in an aquifer with storative semiconfining layers, *Water Resour. Res.*, *21*(8), 1121–1131.

Naff, R. L. (1991), Radial flow in heterogeneous porous media: An analysis of specific discharge, *Water Resour. Res.*, *27*(3), 307–316.

Neuman, S. P., and S. Orr (1993), Prediction of steady state flow in nonuniform geologic media by conditional moments: Exact nonlocal formalism, effective conductivity, and weak approximation, *Water Resour. Res.*, *29*(2), 341–364.

Neuman, S. P., and P. Witherspoon (1969a), Theory of flow in a confined two-aquifer system, *Water Resour. Res.*, *5*(4), 803–816.

Neuman, S. P., and P. Witherspoon (1969b), Applicability of current theories of flow in leaky aquifers, *Water Resour. Res.*, *5*(4), 817–829.

Neuman, S. P., A. Guadagnini, and M. Riva (2004), Type-curve estimation of statistical heterogeneity, *Water Resour. Res.*, *40*, W04201, doi:10.1029/2003WR002405.

Neuman, S. P., A. Blattstein, M. Riva, D. M. Tartakovsky, A. Guadagnini, and T. Ptak (2007), Type curve interpretation of late-time pumping test data in randomly heterogeneous aquifers, *Water Resour. Res.*, *43*, W10421, doi:10.1029/2007WR005871.

Oliver, D. S. (1990), The average process in permeability estimation from well-test data, *SPE Form. Eval.*, *5*(3), 319–324.

Raghavan, R. (2004), A review of applications to constrain pumping test responses to improve on geological description and uncertainty, *Rev. Geophys.*, *42*, RG4001, doi:10.1029/2003RG000142.

Renard, P., and G. de Marsily (1997), Calculating the equivalent permeability: A review, *Adv. Water Resour.*, *20*(5–6), 253–278.

Riva, M., A. Guadagnini, S. P. Neuman, and S. Franzetti (2001), Radial flow in a bounded randomly heterogeneous aquifer, *Transp. Porous Media*, *45*(1), 139–193.

Rubin, Y. (2003), *Applied Stochastic Hydrogeology*, Oxford Univ. Press, New York.

Sanchez-Vila, X. (1997), Radially convergent flow in heterogeneous porous media, *Water Resour. Res.*, *33*(7), 1633–1641.

Sanchez-Vila, X., C. L. Axness, and J. Carrera (1999a), Upscaling transmissivity under radially convergent flow in heterogeneous media, *Water Resour. Res.*, *35*(3), 613–621.

Sanchez-Vila, X., P. M. Meier, and J. Carrera (1999b), Pumping tests in heterogeneous aquifers: An analytical study of what can be obtained from their interpretation using Jacob's method, *Water Resour. Res.*, *35*(4), 943–952.

Sanchez-Vila, X., A. Guadagnini, and J. Carrera (2006), Representative hydraulic conductivities in saturated groundwater flow, *Rev. Geophys.*, *44*(3), RG3002, doi:10.1029/2005RG000169.

Serrano, S. E. (1997), The Theis solution in heterogeneous aquifers, *Ground Water*, *35*(3), 463–467.

Shvidler, M. I. (1962), Filtration flows in heterogeneous media (in Russian), *Izv. Akad. Nauk SSSR Mech.*, *3*, 185–190.

Theis, C. V. (1935), The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage, *Trans., Am. Geophys. Union*, *16*, 519–524.

Trinchero, P., X. Sanchez-Vila, N. K. Copty, and A. N. Findikakis (2008a), A new method to interpret pumping tests in leaky aquifers, *Ground Water*, *46*(1), 133–143.

Trinchero, P., X. Sanchez-Vila, and D. Fernandez-Garcia (2008b), Point-to-point connectivity, an abstract concept or a key issue for risk assessment studies?, *Adv. Water Resour.*, doi:10.1016/j.advwatres.2008.09.001.

Walton, W. (1962), Selected analytical methods for well and aquifer evaluation, in *Illinois State Water Survey Bulletin 49*, 81 pp. Champaign, Ill.

Wu, C.-M., T.-C. J. Yeh, T. H. Lee, N.-S. Hsu, C. H. Chen, and A. F. Sancho (2005), Traditional aquifer tests: Comparing apples to oranges?, *Water Resour. Res.*, *41*(9), W09402, doi:10.1029/2004WR003717.

Yeh, T.-C. J., and S. Liu (2000), Hydraulic tomography: Development of a new aquifer test method, *Water Resour. Res.*, *36*(8), 2095–2105.

Zhu, J., and T. J. Yeh (2005), Characterization of aquifer heterogeneity using transient hydraulic tomography, *Water Resour. Res.*, *41*, W07028, doi:10.1029/2004WR003790.

N. K. Copty and M. S. Sarioglu, Institute of Environmental Sciences, Bogazici University, Bebek, 34342 Istanbul, Turkey. (ncopty@boun.edu.tr)

A. N. Findikakis, Bechtel National Inc., 50 Beale Street, San Francisco, CA 94105-1895, USA.

X. Sanchez-Vila and P. Trinchero, Department of Geotechnical Engineering and Geosciences, Technical University of Catalonia, C/Jordi Girona 1-3, E-08034 Barcelona, Spain.